APPENDIX A

TITLE: SELECTIVE REFLECTING

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THEORY

The sodium doublet consists of two spectral lines in the yellow having wavelengths of 5890 and 5896 Angstroms. The 5890 A line is twice as intense as the 5896 A line.

The Fabry-Perot Optical Resonator

A Fabry-Perot optical resonator is a resonant cavity formed by two parallel reflecting mirrors separated by a medium such as air or gas. A He-Ne laser is basically a Fabry-Perot resonator.

When the mirrors are aligned perfectly parallel to each other, the reflections of the light waves between the two mirrors interfere constructively and destructively, giving rise to a standing wave pattern between the mirror surfaces, just like standing waves on a string. For standing waves, any wavelengths that are not an integer multiple of half a wavelength will interfere destructively. This is shown below in **Figure 1 (b)**. The standing waves therefore must satisfy the condition:

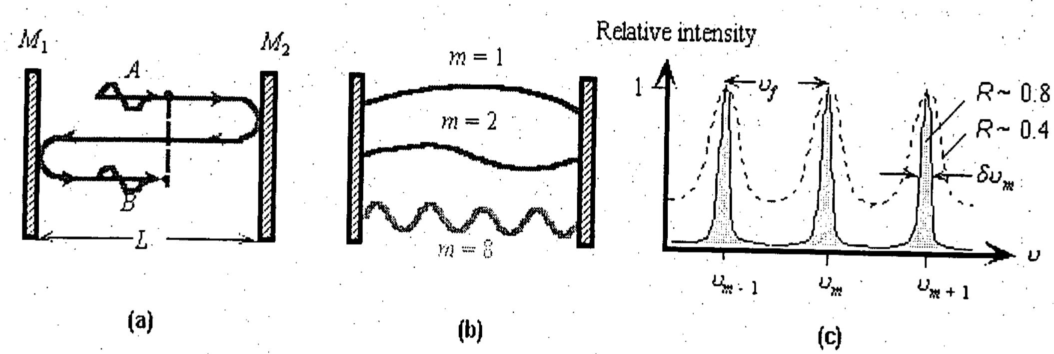
$$m\left(\frac{\lambda}{2}\right) = L$$

where L is the length of the cavity, λ is the wavelength and m is an integer. Expressed in terms of the frequency (ν):

$$\nu_m = m\,(c / 2L)$$

The electric field in the cavity can be calculated by looking at the path of one light ray. In **Figure 1 (a)** a light ray is travelling in one direction at A. It is reflected at the mirror M2 and then again at M1, resulting in a wave (B) that is once again travelling in the same direction as the original ray.

Figure 1 Fabry-Perot Optical Cavity



Schematic illustration of the Fabry-Perot optical cavity and its properties. (a) Reflected waves interfere. (b) Only standing EM waves, *modes*, of certain wavelengths are allowed in the cavity. (c) Intensity vs. frequency for various modes. *R* is mirror reflectance and lower *R* means higher loss from the cavity.

The wave at B now has a different magnitude, that is determined by the reflection coefficients of the mirrors, and also a different phase from the original wave. If we assume the the mirrors have the same reflection coefficient (*r*), and the phase difference is given by ***2kL*** (where ***k*** is the wave number $2\pi/\lambda$), then we can write the sum of the two waves as:

$$A + B = A + Ar^2 e^{-j2nkL}$$

As the wave continues to be reflected, there will be more terms of higher order added to the sum shown above, resulting in a geometric series that can be evaluated as:

$$E_{cavity} = A / (1 - r^2 e^{-j2nkL})$$

The intensity in the cavity is the square of the electric field amplitude. Defining r^2 as the *reflectance* **R**, we can write the intensity in the cavity as:

$$I_{cavity} = \frac{I_0}{(1-R)^2 + 4R\sin^2(nkL)}$$

where $I_0 = A^2$ is the original intensity.

The maximum intensity occurs whenever the $\boldsymbol{sin^2(nkL)}$ term in the denominator is equal to zero, which occurs whenever $\boldsymbol{nkL}$ is an integer multiple of π -- in other words, whenever $\boldsymbol{nkL = m\pi}$. These maxima, of course, correspond to the allowed cavity modes.

Figure 1 (c) shows the intensity pattern in the Fabry-Perot resonator. You can see that the intensity peaks are sharper for higher values of **R**, as can be determined from equation (5). The width of the peaks at FWHM (full width half maximum) is defined as the spectral width $\delta\nu_m$. The separation between the peaks is defined as the mode separation $\Delta\nu_m$. The mode separation is equal to the fundamental mode in the cavity ν_f(for which m = 1). The ratio of the mode separation to the spectral width is called the ***finesse*** (F) of the cavity and is given by:

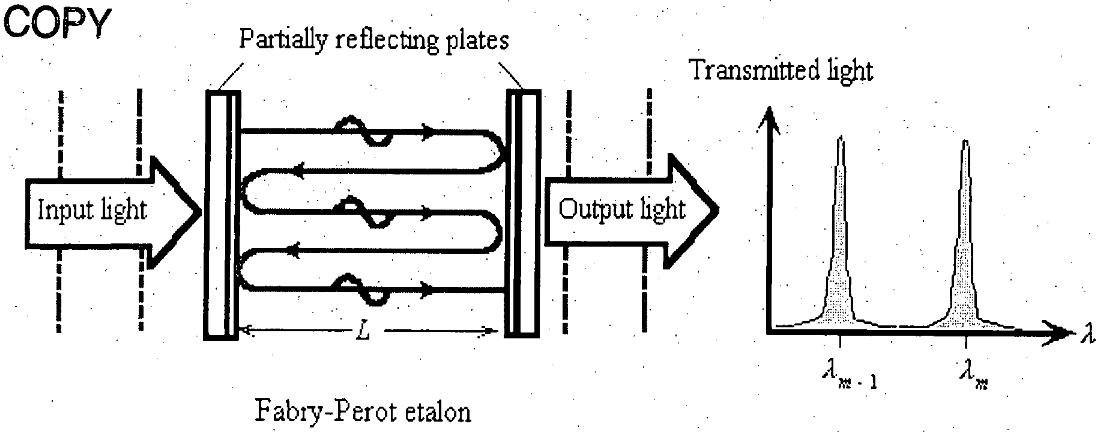
$$F = \nu_f / \delta\nu_m$$

$$= \pi R^{1/2} / (1-R)$$

The ***finesse*** is useful because if gives a quick indicator of the sharpness of the peaks. Larger ***finesses*** lead to sharper peaks.

At each reflection from the end mirrors, some of the light is transmitted, resulting in an output spectrum with intensity peaks at the allowed cavity modes. The output spectrum of the Fabry-Perot resonator is shown in Figure 2 below.

Figure 2 The Output Spectrum of the Fabry-Perot Resonator



Transmitted light through a Fabry-Perot optical cavity.

The output intensity will be a fraction of the input intensity. The amount of transmitted light is determined by the reflectance **R.** A fraction of the incident light (1-R) will enter the cavity as $I_{incident}$, of which part (again 1-R) will leave the cavity as $I_{transmitted}$. Thus the ratio of transmitted to incident light is given by:

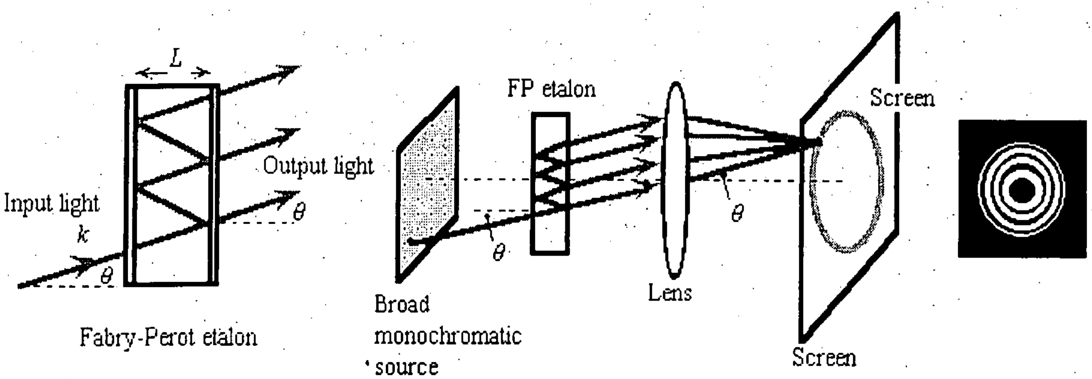
$$\frac{I_{transmitted}}{I_{incident}} = \frac{(1-R)^2}{(1-R)^2 + 4R\sin^2(nkL)}$$

The Fabry-Perot Interferometer

The Fabry-Perot resonator is widely used as a multiple-beam interferometer, an instrument first constructed in the early 1800s by Charles Fabry and Alfred Perot. The Fabry-Perot interferometer has an extremely high resolving power - about 10 times better than a grating spectrometer (which is already at least an order of magnitude better than a prism spectrometer).

A schematic diagram of a Fabry-Perot interferometer is shown in **Figure 3** below.

Figure 3 The Fabry-Perot Interferometer



Fabry-Perot optical resonator and the Fabry-Perot interferometer (schematic)

The interferometer consists of a Fabry-Perot resonator, called an etalon, and a lens to focus the light on a screen or, for the instrument used in the lab, at the observation point (your eye will be at the position of the screen in the diagram).

When a broad monochromatic light source is used as the input to the interferometer, a portion of the light ray entering at an angle θ to the axis normal to the etalon will also leave the etalon, at the same angle. A portion of the light ray will be reflected and then leave the etalon, parallel to the first transmitted ray. This pattern is repeated for multiple reflections. All the rays that are parallel, and in the same plane of incidence, will combine at a point P on the screen. Since the individual rays are not coherent with each other, the intensity at P will simply be the sum of the intensities of the individual waves. The

resulting interference pattern is a series of concentric light and dark rings.

The fringe system of a Fabry-Perot Interferometer is the same as the basic equation for the cavity modes in the resonator, but is generalized to include light rays at an angle θ to the normal. The path of the ray is resolved into components parallel and perpendicular to the normal at the mirror face, so that the parallel component (which contributes to the fringe intensity) is given by $\boldsymbol{k}\cos\theta$, and the resulting equation is:

$$m\lambda = 2nL\cos\theta$$

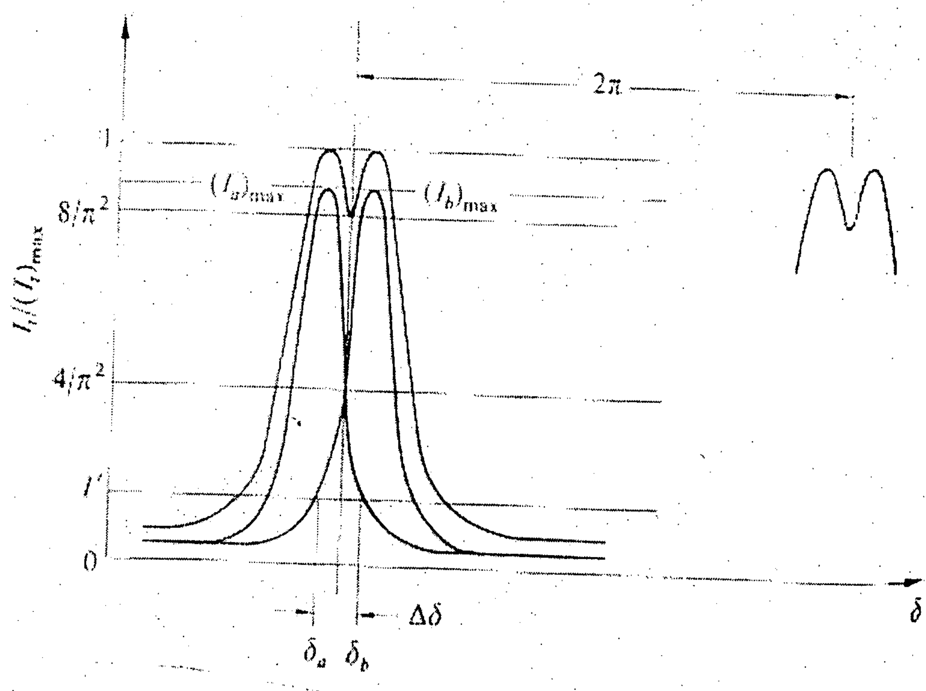
Resolution of the Fabry-Perot interferometer

Rayleigh's criterion establishes that two adjacent interference fringes (or spectral modes) are just resolvable when the combined intensity at the "saddle point" (center between the two maxima) is equal to :

$$I_{center} = I_{max}\,(8/\pi^2)$$

Figure 4 shows an example with two overlapping fringes that have equal maximum intensities.

Figure 4 Intensity pattern from two overlapping fringes
(Hecht, E. ***Optics***, Addison Wesley Longman, Inc., 1998, p. 416)



The peak that results from the addition of the two fringes is given by:

$$(I_1)_{max} = (I_2)_{max} + I'$$

The phase difference between the two maxima can be calculated using the relatiionship above. For large values of the coefficient of finesse (*f*) the phase

difference is approximately:

$$\phi \approx \frac{4.2}{\sqrt{f}}$$

where f = coefficient of finesse = $(2r / (1 - r^2))^2$

This represents the smallest phase increment that separates two peaks that are just barely resolvable. The resolution can then be stated in terms of the wavelength and the *finesse* **F** of the resonator as

$$Resolution = \mathbf{F}\,(2nL / \lambda)$$

Measuring the Sodium Doublet Separation on the Fabry-Perot Interferometer:

The length L of the Fabry-Perot interferometer is adjusted by using a micrometer screw to move one of the parallel mirrors forming the etalon. The mirror position can be read on the micrometer, which is calibrated in millimeters. The mirror is moved by a lever connected to the micrometer screw, so the ratio of the micrometer reading to the actual movement of the mirror is 1:5.

Using a sodium light source, a set of two superimposed fringe patterns from the sodium doublet can be observed. The sodium doublet consists of two spectral lines in the yellow having wavelengths of 5890 and 5896 Angstroms. The 5890 A line is twice as intense as the 5896 *A* line.The movable mirror can be adjusted so that the ultrafine fringes due to the 5896 *A* line will appear to be exactly halfway between the heavier fringes due to the 5890 *A* line.

Then, with air for the medium between the mirrors, we have $n = 1$ and, at the center of the fringe pattern, $\cos\theta = 1$. The fringe system equation becomes:

$$m\lambda = 2L$$

A first micrometer reading is taken:

$$2\,L_1 = m_1\,\lambda_1 = (m_2 + n + 1/2)\,\lambda_2$$

where λ_1 is greater than λ_2. The last term on the right-hand side means that the order of the shorter wavelength ring system must differ from that of the longer wavelength ring system by an odd half integer. This is because the ring patterns have been adjusted to fall midway between each other.

The mirror is then adjusted, and the fringe pattern will seem to move outwards from the center of the pattern. When the fine rings are once again halfway between the heavier rings, a second reading is taken:

$$2\,L_2 = m_2\,\lambda_1 = (m_2 + n + 3/2)\,\lambda_2$$

(Note that if we had started with the plates in contact with each other, the quantity 'n' would not have appeared in the two equations immediately preceding.)

Subtracting these two equations:

$$2\,(L_2 - L_1) = (m_2 - m_1)\,\lambda_1 = (m_2 - m_1 + n + 1)\,\lambda_2$$
$$(m_2 - m_1)\,(\lambda_1 - \lambda_2) = \lambda_2$$
$$(m_2 - m_1) = \lambda_2 / (\lambda_1 - \lambda_2)$$

Since λ_1 and λ_2 are approximately equal:

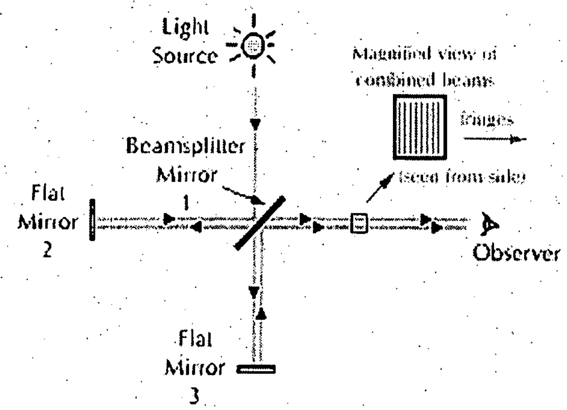
$$\lambda_1 - \lambda_2 = \lambda^2 / 2\,(L_2 - L_1)$$

The separation $(L_2 - L_1)$ is evaluated as: 0.10 $(D_2 - D_1)$ K, where $(D_2 - D_1)$ is the change of the micrometer reading as read in millimeters, and K is the ratio of carriage movement to micrometer reading (K = 0.20).

Finally, the doublet separation is given by:

$\Delta\lambda = 8.68 / \mathbf{(D_2 - D_1)}$ **Angstroms**

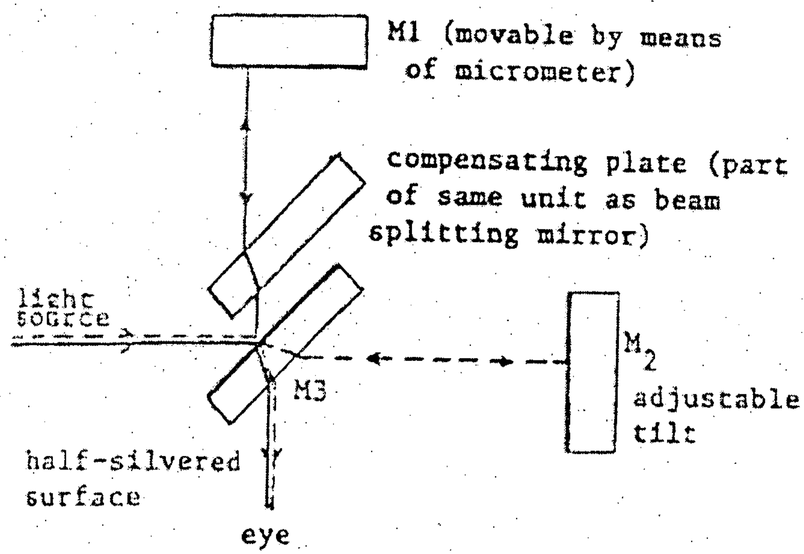
where $(D_2 - D_1)$ is the change of the micrometer reading as read in millimeters



Michelson Interferometer

The Michelson interferometer is one of the most useful of all optical instruments. It was originally designed by Michelson and Morley to detect the "ether" medium in which light waves were supposed to propagate, just as sound waves propagate in air. The negative result of that experiment led Einstein to postulate the special theory of relativity based on the principle that the speed of light is the same in all inertial reference frames. These days, the Michelson interferometer is used for very accurate determinations of the wavelength of spectral lines. In fact, until recently, the meter was defined as 1,650,763.73 times the wavelength of the orange-red spectral line of Krypton-86, and the Michelson interferometer was one of the instruments used by the National Bureau of Standards to measure that wavelength accurately.

The apparatus basically consists of a half-silvered beam-splitting mirror M3 from which half of the light travels to mirror M1 and is reflected, while the other half of the light goes to mirror M2 and is reflected.



The reflected light beams from the two mirrors then recombine at M3 and are examined by eye as shown. Whether the interference between the two beams will be constructive or destructive depends upon the path lengths in the two arms. Notice that movement of mirror M1 by one-half wavelength will cause the beams to undergo a net path difference of one whole wavelength. The purpose of the compensating plate is to ensure that both beams travel through equal path lengths in glass. The compensating plate is exactly equal in thickness to mirror M3. In the diagram shown, you can see that each beam passes through 3 thicknesses of glass in going from the source to your eye.

Mirror M2 has two tilt adjustment screws which can be used to align M2 with mirror M1 mounted on the carriage. The carriage is movable by means of a micrometer screw which actuates a pivoted beam coupled to the carriage. The beam provides a 5:1 reduction from the indicated micrometer reading to the actual length traversed by the carriage. The micrometer itself has 25 mm of movement and vernier graduations for reading to 0.01 mm, hence the carriage has 5 mm movement which can be read to 0.002 mm.

THEORY OF DOUBLE BEAM INTERFERENCE:

1. Interference when light of a single wavelength is used:

Suppose the source produces light waves of a given wavelength. These waves are incident on the beam splitter, and can be written as

$$\begin{aligned} Ein &= A \sin (kx - \omega t - \alpha) \\ &= A \sin (2\pi x/\lambda - 2\pi f\, t - \alpha) \end{aligned}$$

where $k = 2\pi/\lambda$ is the propagation constant, and ω is the angualr frequency.

If we set

$$\phi (t) = \omega t - \alpha$$

$$Ein = A \sin (kx - \phi (t))$$

Let us call the position of the beam-splitting mirror $x = 0$, so

$$Ein = A \sin (- \phi (t))$$

When the incident light splits, half goes a distance $2l_1$, the other half a distance $2l_2$ and the two returned waves are

$$E_1 = a \sin (2kl_1 - \phi (t) - \pi)$$

$$E_2 = a \sin (2kl_2 - \phi (t))$$

The difference of phase of π between the two returned waves arises because half of the incident beam reflects externally from the beam-splitting mirror (after travelling to M1) while the other half reflects internally at the beam-splitting mirror (after travelling to M2). In the first case, the beam is travelling in air, and reflecting at the air/glass interface; in the second case, the beam is travelling in glass and reflecting at the galss/air interface.

This difference in boundary conditions leas to the phase difference of π.

The sum of superpositon of these two waves is what we view. If we use the identity

$$\sin a + \sin b = 2 \cos [(a-b)/2] \sin [(a+b)/2]$$

we obtain

$$Eout = E_1 + E_2 = 2a \cos [k(l_1 - l_2) - \pi/2) \sin [k(l_1 + l_2) - \pi/2\ \phi (t))$$

The eye detects the intensity of the wave, which is proportional to the time average of the square of the electric field Eout

$$I \propto E^2 = 4a^2 \cos^2 [k(l_1 - l_2) - \pi/2)] \sin [k(l_1 + l_2) - \pi/2\ \phi (t))$$

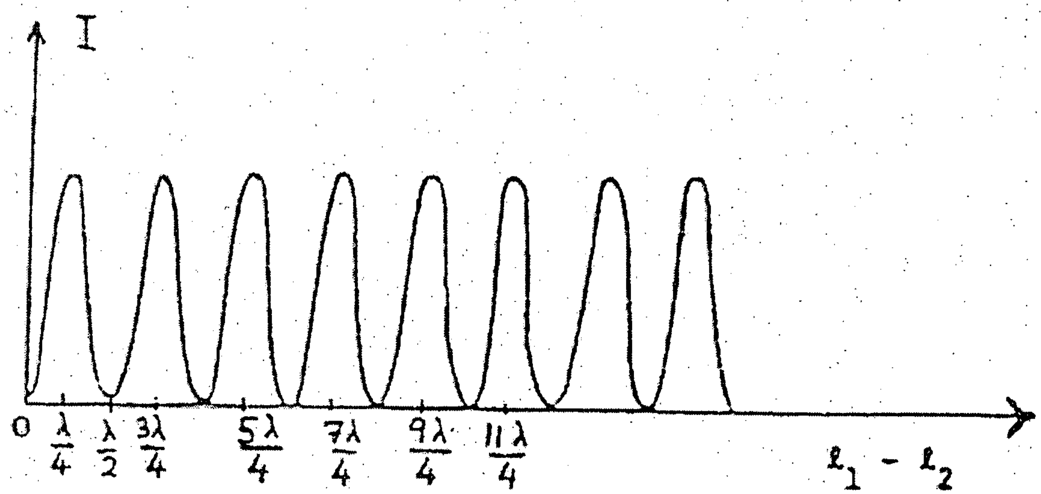
In the time average, only the last term on the right enters, and since the time average of $\sin^2$ is 1/2

$$I \propto 4a^2 \cos^2 [k(l_1 - l_2) - \pi/2)] (1/2) = 2a^2 \sin^2 [k(l_1 - l_2)]$$

where we have also used $\cos(\theta-\pi/2) = \sin \theta$

The maximua of I thus occur when $\sin [k(l_1 - l_2)] = +1$. Since $k= 2\pi/\lambda$:

Maxima: $\lambda / 4, 3\lambda / 4, 5\lambda / 4, \ldots$



We see that movement of M1 by $\lambda / 2$ causes one complete interference fringe to pass by. Thus, by counting the number of fringes that pass by when the micrometer screw changes l_1 by a given amount, we can determine the wavelength of the light used.

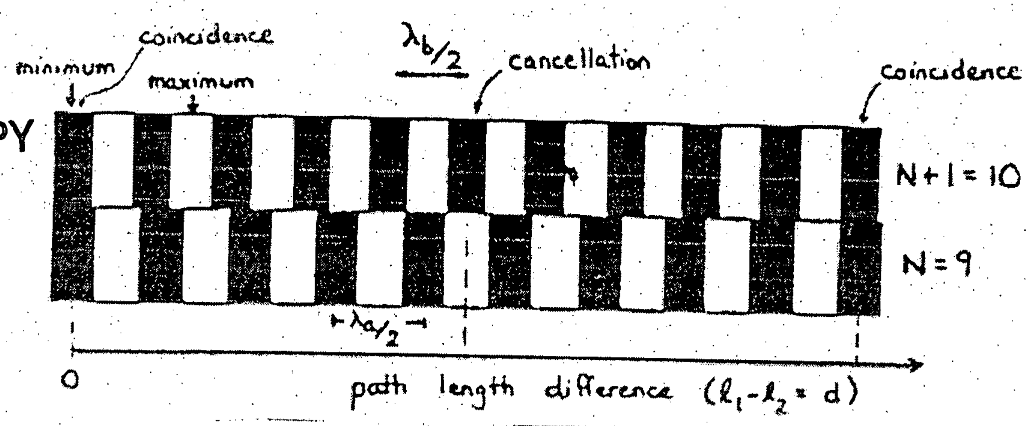
2. Interference when the incident light consists of two closely separated wavelengths:

$$E_{in} = A \sin [k_A x - g(t)] + B \sin [k_B x - h(t)]$$

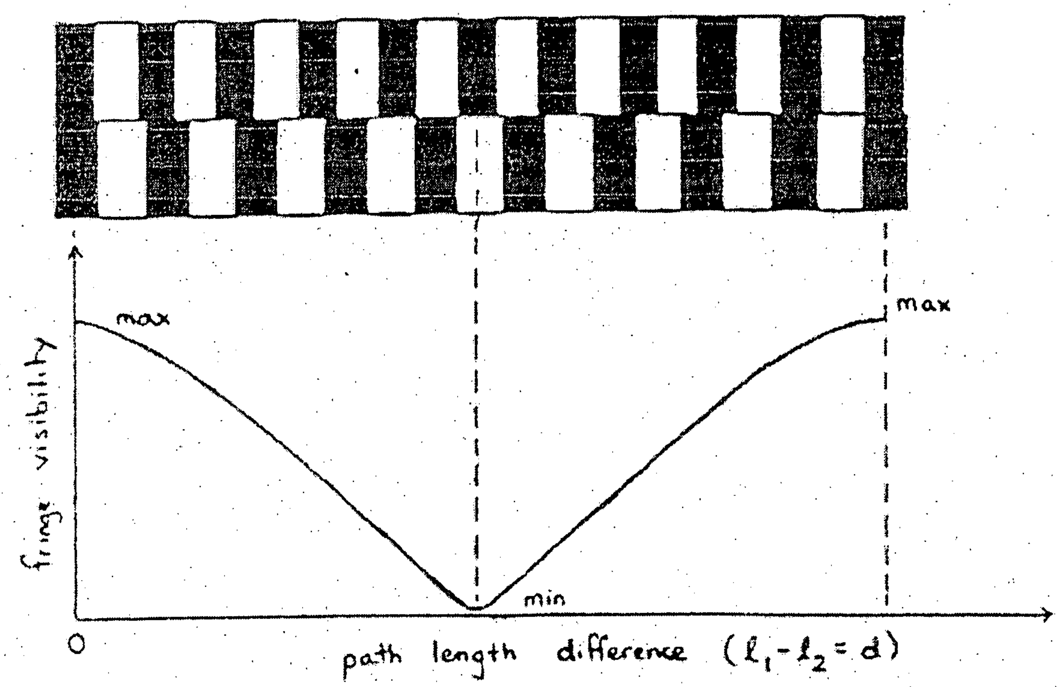
In the incoming light, there are two wavelengths, λ_A and λ_B of amplitudes A and B. (In the following descussion it will be asumed that the amplitudes of these two waves are approximately equal.) It is very important to realize that the time terms g(t) and h(t) are random with respect to one another. these wavelengths arise when an outer electron of an atom jumps from a higher energy level to a lower level. The two energy levels involved in producing the first wavelength are different from the two levels twhich give the second eavelength, and the electron transitions between the first two levels are independent of the transitions between the other two levels because the jumps occur in different atoms. Hence, there is no relationship in time between the appearance of the two waves, i.e. they are random in time with respect to each other (incoherent).

Because of the incoherence of the two wavelengths, their behavior in the Michelson interfereometer must be treated individually (i.e. there can be no "interference" between the two waves of different wavelength). thus, each wavelength has its own intensity pattern in the interferometer, as described in the previous section.

The two intensity patterns in the interferometer, arising from the two wavelengths, can be represented as a function of mirror movement by the following bar diagram:



The two patterns coincide for $l_1 - l_2 = d = 0$, cancel each otheer as the mirror is moved from zero path difference, and coincide again as the mirror is moved further. This coincidence and cancellation continues as the mirror is moved. The overall fringe visibility will thus vary with mirror position. The fringes will be very distinct (minima black, maxima bright) when the patterns coincide, and the individual fringes will fade out into the background when the patterns cancel. This variation is shown in the diagram below.



Recall that the distance the mirror must be moved between consecutive fringe maxima is $\lambda/2$. Also, note from the previous bar diagram, that if the number of fringe maxima between coincidences of the two intensity patterns is N for λ_A, then it is N+1 for λ_B.

Let d_c be the distance the mirror must be moved between consecutive positions of pattern coincidence.

$$dc = (N+1)(\lambda_B/2) = N(\lambda_A/2)$$
$$N\,\lambda_B/2 + \lambda_B/2 = N(\lambda_A/2)$$
$$\lambda_B = N(\lambda_A - \lambda_B)$$

Let $\Delta\lambda = \lambda_A - \lambda_B$

and $\lambda_{AVE} = N\Delta\lambda$

$$N = \lambda_{AVE} / \Delta\lambda$$

Therefore,

$$d_c = N(\lambda_A/2) = (\lambda_{AVE} / \Delta\lambda)(\lambda_{AVE} / 2)$$

$$\mathbf{d_c = \lambda^2_{AVE} / 2\Delta\lambda}$$

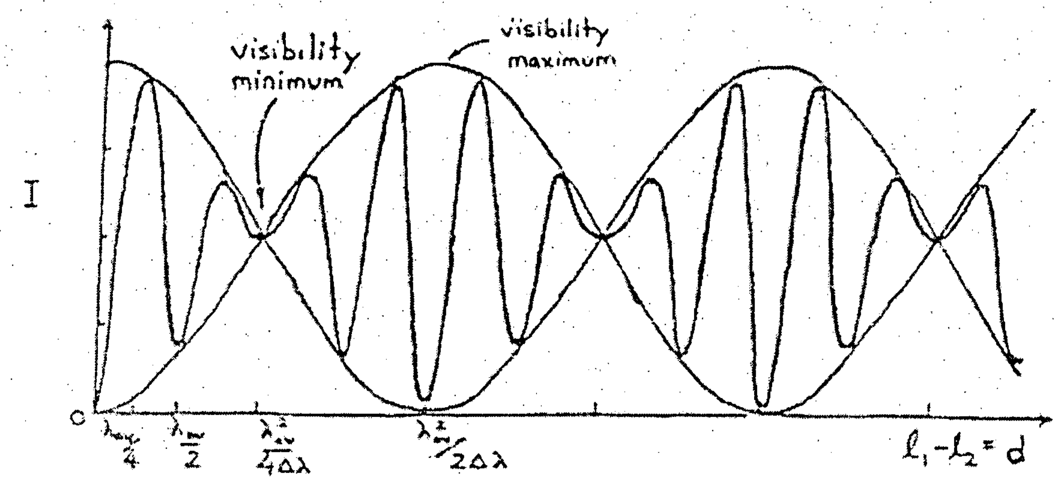
where d_c is the distance the mirror moves (Ratio of mirror : micrometer movement = 1 : 5)
and λ_{AVE} = 5893 Angstroms

Finally, the doublet separation is given by:

$\Delta\lambda = 8.68 / \mathbf{(D_2 - D_1)}$ **Angstroms**

where $(D_2 - D_1)$ is the change of the micrometer reading as read in millimeters

The overall pattern observed in the interferometer is shown roughly in the following diagram:



By determining the mirror movement between the individual fringes, the average wavelength can be calculated. by determining the mirror movement between two successive visibility maxima (positions of coincidence) the wavelength difference between the two wavelengths can be calculated.. Note that the wavelength difference can also be obtained from the mirror movement between two successive visibility minima (positions of cancellation where the individual fringes disappear into the background light) since

d (coincidence) = d (cancellation).